Application of Non-divergent Barotropic Model to Predict Flow Patterns in the Indian Region

By J. Shukla and K. R. Saha

Institute of Tropical Meteorology, Poona-5, India
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Abstract

The barotropic model with the non-divergent stream function derived from wind analysis as input is used for predicting 500 mb level flow patterns in the Indian region. Appropriate boundary conditions for computation of the stream function from the observed wind information are prescribed. Such boundary conditions are considered suitable for the purpose which maximise kinetic energy in the reconstructed wind and minimise root-mean-square-vector-error between the reconstructed wind and the observed wind.

Using data of a monsoon situation in June 1966, in which a special interest is movement of a monsoon depression from the Head of the Bay of Bengal in a northerly direction, integration is performed in one-hour time step upto a period of 48-hrs. The 24-hrs. forecasts of the flow patterns appear to compare favourably with the observed flow patterns while the 48-hrs. forecasts appear to show considerable departure from the observed patterns. The r. m. s. v. e. between the forecast and observed values of the non-divergent component of the winds for the Indian region are found to be 3.5 m/s for 24-hrs. forecasts, and 6.0 m/s for 48/hrs. forecasts. The mean vector error between the predicted and the observed positions of the center of the deep depression is 140 km for 24-hrs. forecasts, and 205 km for 48-hrs. forecasts.

1. Introduction

It is well-known that in the Tropics, except in well-defined synoptic systems such as depressions and cyclones, the temperature and pressure gradients are generally weak and the relationship between wind and pressure is much more complex than that given by a simple geostrophic relationship. In addition there are wide gaps in radiosonde observational network especially over the oceanic regions and in many parts available observations suffer from instrumental and other errors especially over the Indian region (Ananthakrishnan *et al.*, 1966, 1967). For these reasons, it is now largely accepted that in low latitudes the flow-patterns depicted by the winds are more reliable than those deduced from contour analysis (Palmer, 1952; La Seur, 1960; Miyakoda, 1960a; Yanai and Nitta, 1967). It was, therefore, thought that it would be more appropriate to use wind information as the basic input in a prediction model than contour height information. The barotropic model using the stream function derived from objective wind analysis as input has recently been employed by Vederman, Hirata and Manning (1966) to predict flow-patterns at several levels in the Tropical Pacific. In the present paper the same basic model has been applied to the Indian region. However, as shown by Hawkins and Rosenthal (1965) the successful derivation of a stream function field from the observed wind information depends very much on the boundary conditions used to compute the field. Hence, in the present paper effort has been made through an iterative procedure to arrive at such boundary condition and initial stream function field which are likely to ensure a realistic representation of the flow patterns. Keeping in view that the computed stream function is to be used as input in a dynamical prediction model we have assumed that a stream function field which maximises kinetic energy in the reconstructed wind and minimises the root-mean-square-vector-error (hereafter called r. m. s. v. e.) between the reconstructed wind and the observed wind is appropriate for the purpose. However, in cases when both the criteria are not satisfied simultaneously, weightage may be given to the former.

To test the model, forecasts of flow patterns at 500 mb level in the Indian region were obtained for 24 hours on four consecutive days and 48 hours on two days in June. 1966. The results and the verification of the forecasts are discussed in Section 4.

2. Computation of the initial stream function field

From the Helmholtz theorem

$$\vec{V}=\vec{k}\times\nabla\psi+\nabla\chi \tag{1}$$

which expresses the horizontal wind vector $\vec{V}$ in terms of the non-divergent stream function ψ and the irrotational velocity potential χ we obtain the following expresssions for relative vorticity ζ and divergence D

$$\zeta=\vec{k}\cdot\nabla\times\vec{V}=\nabla^2\psi=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \tag{2}$$

$$D=\nabla\cdot\vec{V}=\nabla^2\chi=\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \tag{3}$$

where u and v are the zonal and the meridional components of $\vec{V}$, positive towards the east (x) and the north (y) respectively.

Equations (2) and (3) may be solved by method of relaxation to obtain the ψ and χ fields under suitable boundary conditions. Sangster (1960), Rosenthal (1963) and Hawkins and Rosenthal (1965) have given details of the procedure adopted by them to compute values of ψ at the boundary with the help of the Helmholtz theorem. In the present paper, we have proceeded on somewhat similar lines and used a scheme which maximises kinetic energy in the non-divergent component of the wind and minimises the r. m. s. v. e. between the reconstructed non-divergent component of the wind and the observed wind. The scheme may be explained as follows: If s and n are distances on the earth along the boundary (positive in the anti-clockwise sense) and normal to the boundary (positive in the direction of the outward normal) respectively, we have from equaton (1)

$$\frac{\partial\psi}{\partial s}=-v_n+\frac{\partial\chi}{\partial n} \tag{4}$$

$$\frac{\partial\chi}{\partial s}=v_s-\frac{\partial\psi}{\partial n} \tag{5}$$

where v_s and v_n are the observed wind components in s and n directions respectively. To start with, equation (3) is solved with $\chi=0$ at the boundary. The values thus derived at the interior grid-points are used in equation (4) to obtain new values of ψ at the boundary by integrating equation (4) along the boundary. The normal derivative $\frac{\partial\psi}{\partial n}$ or $\frac{\partial\chi}{\partial n}$ is calculated by linear extrapolation of the derivative from interior grid-points to the boundary.

After the boundary values of ψ is determined in this manner, equation (2) is solved by accelerated Liebmann relaxation method using an over-relaxation coefficient of 0.7 to obtain ψ field.

Table 1. Percentage kinetic energy and the r. m. s. v. e. for reconstructed wind field

Date	Percentage K. E. $\vec{V}=\vec{k}\times\nabla\psi$	Percentage K. E. $\vec{V}=\vec{k}\times\nabla\psi+\Delta\chi$	r.m.s.v.e. (meters/sec) $\vec{V}=\vec{k}\times\nabla\psi$	r.m.s.v.e. (meters/sec) $\vec{V}=\vec{k}\times\nabla\psi+\Delta\chi$
14. 6. 66	74. 0	86. 0	2. 8	0. 999
15. 6. 66	72. 0	83. 0	3. 27	1. 75
16. 6. 66	68. 0	80. 0	3. 31	1. 5
17. 6. 66	73. 0	83. 0	2. 69	1. 52

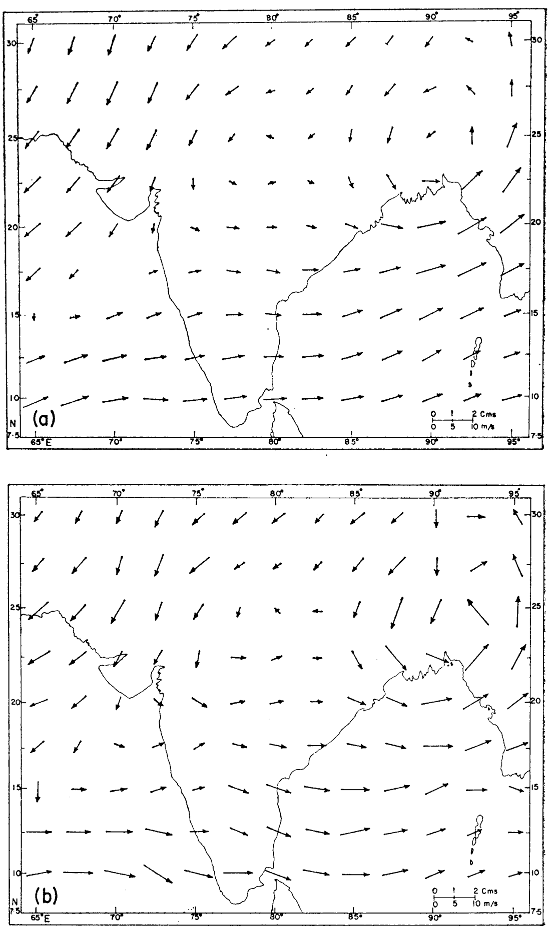

Fig. 1. Observed and reconstructed wind fields at 00Z on 17 June, 1966. (a) observed, (b) reconstructed (non-divergent)

This same procedure is repeated, using newly obtained ψ, to calculate a new χ field using equation (3) and (5) which in turn is used to calculate a new ψ field using equations (2) and (4). This procedure is repeated and from the successive computations a stream function field is finally selected on the basis of maximisation of K. E. contained in ψ and minimisation of r. m. s. v. e. between reconstructed non-divergent wind and the original wind. It may be seen that the first stage of the above iteration corresponds to Sangster's method of calculating ψ. Actual computations also revealed that in some cases the ψ corresponding to Sangster's method itself meets with the aforesaid criteria. However, in some cases the criteria is met in subsequent stages.

Table 1 gives the percentage kinetic energy and the r. m. s. v. e. between the observed and the reconstructed winds on 14-17 June 1966.

Fig. 1 (a) and (b) show the observed winds and the reconstructed non-divergent winds respectively on 17 June 1966. It may be seen that the two fields compare fairly well with each other.

3. The prediction model and the computational scheme

In the case of non-divergent barotropic atmosphere, the prediction equation using the stream function is given by

$$\nabla^2\left(\frac{\partial\psi}{\partial t}\right)=J(\nabla^2\psi+f,\ \psi) \qquad (7)$$

where $(\nabla^2\psi+f)$ is the absolute vorticity, f being the coriolis parameter. Equation (7) can be solved numerically by the relaxation technique using suitable finite difference forms for the Laplacian and the Jacobian and prescribing a suitable boundary condition for $\frac{\partial\psi}{\partial t}$. The finite difference forms of the Laplacian and the Jacobian which are used in the present study are the nine point Laplacian suggested by Miyakoda (1960b) and the nine point Jacobian proposed by Arakawa (1963). These may be stated for Laplacian and Jacobian at a point 0 (see Fig. 2) as follows:

$$\nabla^2\psi\equiv(2\,{}^{+}\nabla^2\psi+{}^{\times}\nabla^2\psi)/3$$

where

$${}^{+}\nabla^2\psi=\frac{\psi_1+\psi_2+\psi_3+\psi_4-4\psi_0}{d^2}$$

$${}^{\times}\Delta^2\psi=\frac{\psi_5+\psi_6+\psi_7+\psi_8-4\psi_0}{2d^2}$$

and

$$J(A,\ B)=J^{++}+J^{\times+}+J^{+\times})/3$$

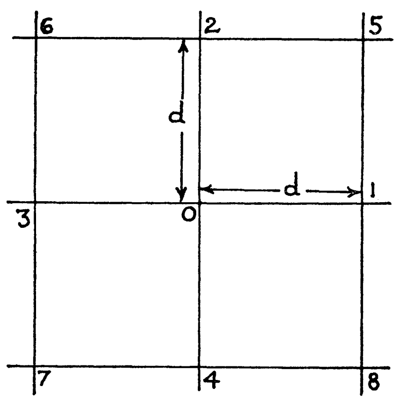

Fig. 2. Arrangement and numbering of points in a horizontal finite difference grid

where

$$J^{++}\equiv[(A_1-A_3)(B_2-B_4)-(A_2-A_4)(B_1-B_3)]/4d^2$$

$$J^{\times+}\equiv[A_1(B_5-B_8)-A_3(B_6-B_7)-A_2(B_5-B_6)+A_4(B_8-B_7)]/4d^2$$

$$J^{+\times}\equiv[A_5(B_2-B_1)-A_7(B_3-B_4)-A_6(B_3-B_2)-A_8(B_4-B_1)]/4d^2$$

where d is the grid length. To perform the integration, $\frac{\partial\psi}{\partial t}$ is kept zero at the boundary and a suitable time increment, consistent with the grid-length is chosen to maintain computational stability. For a grid-length of 2.5° longitude at the equator, a suitable time step is about one hour. The forward difference scheme is used at the first time step and the centered difference scheme at the subsequent time steps. The relaxation is performed by the accelerated Liebmann relaxation technique and the optimum value of the over-relaxation coefficient is 0.7.

4. The computer programme and the forecast

The computational schemes and the forecast model outlined in Sections 2 and 3 have been applied to predict flow patterns at 500 mb level over an area bounded by latitudes 2.5°N and 40°N and longitudes 50°E and 100°E upto 24 hours on four consecutive days viz. 15, 16, 17, 18 June and upto 48 hours on two consecutive days viz. 15 and 16 June 1966. The latitude-longitude intersections at 2.5° intervals constitute the gridpoints, hence the network consists of 21 × 16 grid points. From manually-analysed streamline isotach charts on mercator projection at 00Z map time on the above days, wind direction and speed at the grid points were picked up to serve as basic material for computation of the input stream function. The procedure outlined in Section 2 was followed to obtain the input stream function field. The time step used is one hour and the integration is carried out for 24 and 48 time steps. Computation remained stable throughout the forecast period. All computations were performed on CDC-3600 computer available at the Tata Institute of Fundamental Research, Bombay. The computer output consisted of the forecast stream function and absolute vorticity fields for 24-hours and 48-hours for all the days. However, for economy of space only five computer output charts consisting of one 24-hour and one 48-hour

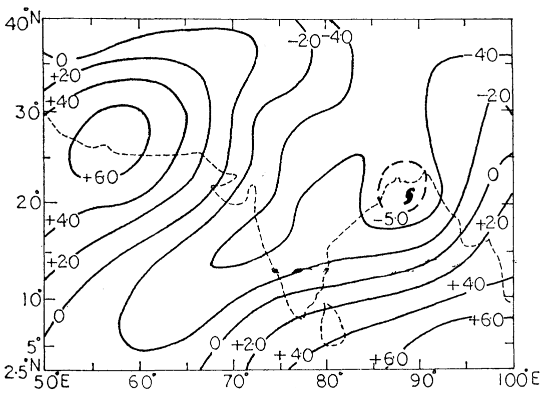

Fig. 3. Observed stream function field ($\times 10^{10}$ cm^2 sec^{-1}) on 16 June (00Z)

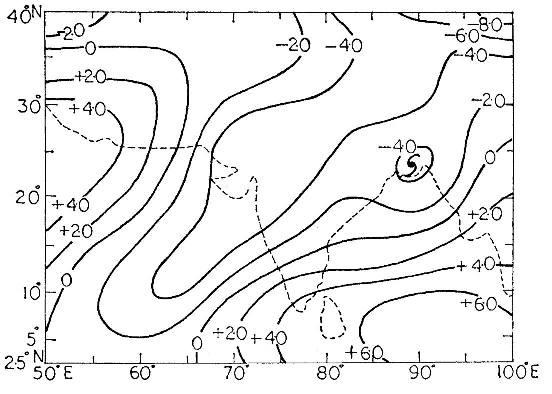

Fig. 4. 24 hr. forecast stream function field ($\times 10^{10}$ cm^2 sec^{-1}) on 17 June (00Z)

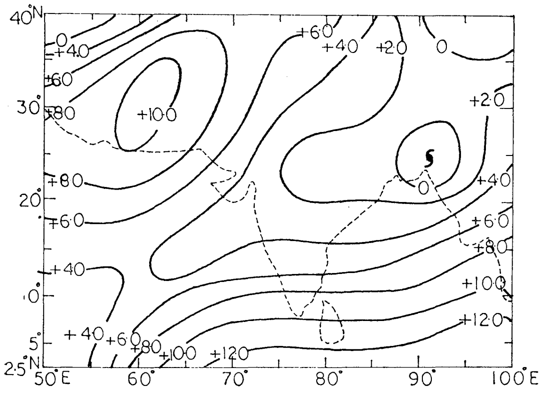

Fig. 5. Observed stream function field ($\times 10^{10}$ cm^2 sec^{-1}) on 17 June (00Z)

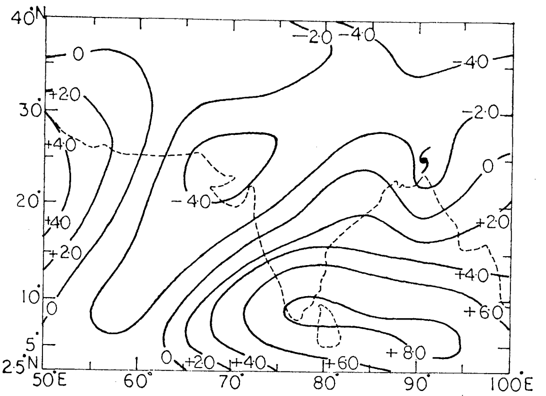

Fig. 6. 48 hr. forecast stream function field ($\times 10^{10}$ cm^2 sec^{-1}) on 18 June (00Z)

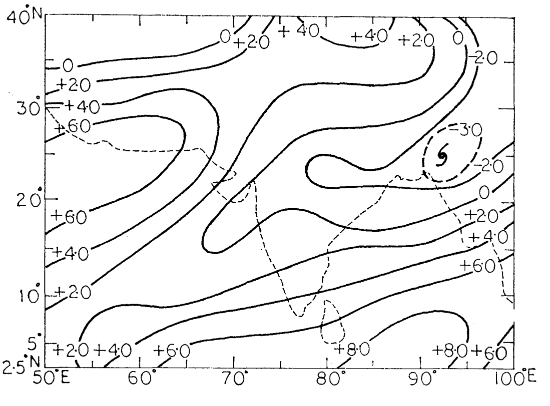

Fig. 7. Observed stream function field ($\times 10^{10}$ cm^2 sec^{-1}) on 18 June (00Z)

Table 2. R.m.s.v.e. between observed and forceast nondivergent component of the wind

Date	R.m.s.v.e. for 24-hours (m/s)	R.m.s.v.e. for 48-hours (m/s)
15	—	—
16	3.4	—
17	3.7	6.0
18	3.1	6.3

Table 3. Forecast and observed positions of the center of the depression

Date	24 hr. forecast position	48 hr. forecast position	Observed	24-hr. forecast vector error (KM)	48-hr. forecast vector error (KM)
15.6	—	—	18.5°N, 91.0°E	—	—
16.6	20.5°N, 90°E	—	22°N, 90.0°E	150	—
17.6	23.0°N, 89.5°E	20.5°N, 92.0°E	24.0°N, 91.0°E	180	360
18.6	25.0°N, 91°E	25.0°N, 91.5°E	24.5°N, 91.5°E	70	50

forecasts and corresponding charts of observed stream function fields are presented in Figs. 3-7. An approximate sketch of India has been included in the charts. The r. m. s. v. e. between the predicted and the observed ψ component of the wind for the Indian region bounded by 7.5°N—27.5°N and 70°E—90°E is given in Table 2.

As mentioned earlier, a special interest of the synoptic situation to which the barotropic model was applied was the movement of a deep depression which was situated over the Head of the Bay of Bengal with center near latitude 18.5°N, longitude 91.0°E on 15 June 1966. In the course of the subsequent two days, the depression moved in a northerly direction. The position of the center of the depression in the ψ field is determined by the position of the minimum value of the stream function.

Table 3 gives the forecast and the observed positions of the center of the depression along with the vector errors on 16, 17 and 18 June for 24 hours forecasts and on 17 and 18 June for 48 hours forecasts.

Outlook

The results of the present study are encouraging. They seem to suggest that the barotropic model with the wind information as input may serve as a suitable operational numerical weather prediction model for the low latitudes in general and the Indian region in particular, provided a suitable objective wind analysis scheme is evolved. It is proposed to test the model with the objectively analysed wind information and extend the computational area and carry out the integration for longer period.

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非発散バロトロピック・モデルによる印度地域の風の場の予報

J. シュクラ・K. R. サハ
熱帯気象研究所（プーナ，印度）

実測の風から導いた流線函数をバロトロピック・モデルに適用して，印度地域の 500 mb 面の風の場の予報を試みた．流線函数を求める際の境界条件は，流線函数から得られる風の運動エネルギーを極大にし，かつ実測の風と流線函数から求められた風との間のルート・ミィーン・スクエヤ・ベクトル・エラーを最小にするに適したものを用いた．

このテストでは，特にモンスーン・低気圧の移動がどれ位予報されるかに焦点をおいて48時間予報を試みた．流れの場の24時間予報は実測の風の場と比較してかなりよく予報されているが，48時間予報は実測値に較べてかなりのずれがあった．風の非発散部分の実測値と予報値との間のルート・ミィーン・スクエヤ・ベクトル・エラーは24時間予報に対しては，3.5 m/sec，48時間予報に対しては 6.0 m/sec であった．モンスーン低気圧の中心の位置の予報値の実測値からの平均のはずれは 24 時間予報で 140 km，48時間予報では 205 km であった．